

1. (a) Differentiate $y = \sqrt{6 - x^2}$ with respect to x . 2

(b) Hence, or otherwise, find $\int \frac{3x}{\sqrt{6 - x^2}} dx$. 2

2. Consider the function defined by $f(x) = x^3 - 6x^2 + 9x + 3$.

(a) Find $f'(x)$. 1

(b) Find the coordinates of the two stationary points and determine their nature 3

(c) Sketch the curve $y = f(x)$ for $0 \leq x \leq 5$ showing all key features 2

(d) Apply the trapezoidal rule with 6 functions values to find an approximation of the area between $f(x) = x^3 - 6x^2 + 9x + 3$ and the x -axis between $x = 0$ and $x = 5$. 2

(e) Find the exact size of the area in part (d). 2

3. The amount of an antibiotic, R , in the human body decays according to the equation:

$$\frac{dR}{dt} = -0.24R$$

where R is measured in mg and t is time in hours.

(a) Show that $R = Ae^{-0.24t}$ is a solution to $\frac{dR}{dt} = -0.24R$ where A is a constant. 1

When $t = 0$, there are 300 mg of the antibiotic in Samantha's body.

(b) Find the value of A 1

(c) What is the amount of the antibiotic in Samantha's body after 16 hours. 1

4. Sarah took out a loan of \$100,000. She plans to repay the loan in equal monthly repayments of \$ M . Interest is charged at a rate of 0.5% per month.
- (a) Show that the amount owing after 3 months is: 1
- $$A_3 = 100\,000 \times 1.005^3 - M(1 + 1.005 + 1.005^2)$$
- (b) If Sarah wishes to pay off the loan by the end of 10 years, then show she will need to pay \$1110 per month. 2
- (c) Show that the amount owing after n months can be written as, 1
- $$A_n = 1.005^n(100\,000 - 200M) + 200M$$
- (d) If Sarah decides that she can only repay \$750 each month, how long will it take her to repay the loan? (Leave your answer to the nearest month) 2
5. Differentiate:
- (a) $\sin(x^2 + 1)$ 2
- (b) $e^x \cdot x^2$ 2
- (c) $(4x^6 + 5)^3$ 2
6. The quadratic equation $2x^2 - 12x + 6 = 0$ has roots α and β .
- (a) Find $\alpha^2 + \beta^2$ 2
- (b) Hence or otherwise, find $\alpha^3 + \beta^3$ 2
7. Find values of m such that $mx^2 + 9x - 1 < 0$ for all x . 2

8. Solve for x : $\log_4 x + \log_4(x - 12) = 3$, for $x > 0$ 2

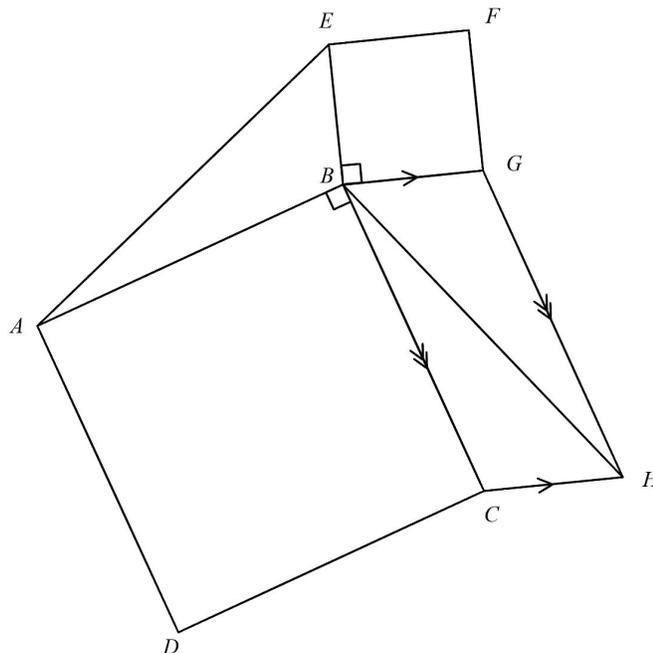
9. Tom and Ali are playing a game against each other. The game is played in rounds. The probability that Tom wins a round is $\frac{3}{5}$. The first player to win 2 rounds wins the whole game.

(a) Find the probability that the game ends after the second round. 1

(b) Find the probability that Tom wins the whole game. 2

(c) Find the probability that the person who wins the first round goes on to win the whole game. 2

10.



In the diagram, $ABCD$ and $BEFG$ are both squares and $BGHC$ is a parallelogram.

By proving a pair of congruent triangles, prove that $AE = BH$. 4