Solutions 3unit

Q1 D

$\alpha+\beta+\gamma = -17$

$-20-1+4 = -17$

Using sum of roots

$\alpha\beta\gamma = 80$

$-20\times -1\times 4 = 80$

using product of roots

Q2 Derivative $= vu' + uv'$ Product rule

$v = x \qquad u = \sin^{-1}(ex)$

$v' = 1 \qquad u' = \dfrac{f'(x)}{\sqrt{1-f(x)^2}}$

$= \dfrac{e}{\sqrt{1-e^2x^2}}$

It's a good idea to write this out.

$$vu' + uv' = \frac{ex}{\sqrt{1-e^2x^2}} + \sin^{-1}(ex)$$

Q3 $u = \sqrt{x} \rightarrow u = x^{\frac{1}{2}}$

$\dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$

$= \dfrac{1}{2\sqrt{x}}$

$2\sqrt{x} \times du = dx$

$\therefore \displaystyle\int \frac{2\sqrt{x}}{(1+x)\sqrt{x}} \cdot du$

It's usually best to first sub out the dx first

$= \displaystyle\int \frac{2}{1+u^2} \cdot du$

you'll need to be confident in recognising integrals like this.

$= 2\left[\tan^{-1}(u)\right]$

$= 2\left[\tan^{-1}(\sqrt{x})\right]_1^3$

Sub the $\sqrt{x}$ back in. Now we can put the boundaries back in.

$= 2\left[\tan^{-1}\sqrt{3} - \tan^{-1} 1\right]$

$= 2\left[\frac{\pi}{3} - \frac{\pi}{4}\right]$

$= \dfrac{\pi}{6}$

Q4 $3^n + 7 < 4^n \qquad n \geq 3$

Step 1: Prove true for $n=3$

$3^3 + 7 < 4^3$

$34 < 64 \quad$ TRUE

Step 2: Assume true for

$3^k + 7 < 4^k$

Step 3: Prove true for

$3^{k+1} + 7 < 4^{k+1}$

cont. above

We are going to fiddle with the assumption until it looks like what we are trying to prove.

$3^k + 7 < 4^k$

$3^{k+1} + 21 < 3\times 4^k$ multiply both sides by 3

$3^{k+1} + 7 + 14 < 3\times 4^k$ split the 21 into 7 + 14

$3^{k+1} + 7 < 3\times 4^k$ we can get rid of the +14 as the LHS will still be less than RHS

$3^{k+1} + 7 < 4\times 4^k$ Add 4^k to RHS

$3^{k+1} + 7 < 4^{k+1}$ TA DA!

Step 4: Statement

Therefore true for $n=k+1$. Since true for $n=3$, it is therefore true of $n=4$, $n=5$ and so on for all $n \geq 3$

Q5 $P(1) = 6$

$6 = 1^3 + 2a - b$

$5 = 2a - b$

$b = 2a - 5$

$P(-1) = 4$

$4 = (-1)^3 - 2a - b$

$b = -2a - 5$

Sub out for b:

$2a - 5 = -2a - 5$

$4a = 0$

$a = 0$

$\therefore b = -5$

Sub $a=0$ into one of the equations above to find b.

Q6 $\dfrac{2x(x-4)}{x-1} \leq 7$ multiply both sides by $(x-1)^2$

$(x-1)(2x)(x-4) \leq 7(x-1)^2$

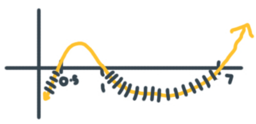
$(x-1)(2x)(x-4) - 7(x-1)^2 \leq 0$ Move everything over to LHS and factorise the $(x-1)$

$(x-1)\left(2x(x-4) - 7x + 7\right) \leq 0$

$(x-1)\left(2x^2 - 8x - 7x + 7\right) \leq 0$

$(x-1)(2x^2 - 15x + 7) \leq 0$

$(x-1)(2x-1)(x-7) \leq 0$



You could draw a table of values to test, but a graph is usually more efficient!

$x \leq \frac{1}{2} \qquad 1 < x \leq 7$

remember $x \neq 1$ as $x-1$ is the denominator in the Q.

Q7 $y = 1 + 2\sin^{-1}(3x-1)$

domain:
$$-1 \leqslant 3x-1 \leqslant 1$$
$$0 \leqslant 3x \leqslant 2$$
$$0 \leqslant x \leqslant \frac{2}{3}$$

$\sin^{-1}$ ◎
$-1 \leqslant$ ◎ $\leqslant 1$

$$y-1 = 2\sin^{-1}(3x-1)$$
$$\frac{y-1}{2} = \sin^{-1}(3x-1)$$

$\therefore$ range: $-\frac{\pi}{2} \leqslant \frac{y-1}{2} \leqslant \frac{\pi}{2}$

$$-\pi \leqslant y-1 \leqslant \pi$$
$$1-\pi \leqslant y \leqslant 1+\pi$$

*You could also find the range by subbing the domain values back into the equation

$\therefore$ $a = 1+\pi$, $b = \frac{2}{3}$, $c = 1-\pi$

Q8

a) $P(\text{all correct}) = \binom{6}{6} 0.25^6$
$$= 2.44 \times 10^{-4}$$

b) $P(\text{4 or more}) = \binom{6}{4} 0.25^4 \times 0.75^2 + \binom{6}{5} 0.25^5 \times 0.75^1 + \binom{6}{6} 0.25^6$
$$= \frac{77}{2048}$$

c) $P(\text{at least } 2) = 1 - P(\text{all wrong} + 1 \text{ correct})$
$$= 1 - \left(\binom{6}{0} 0.75^6 + \binom{6}{1} 0.25^1 \times 0.75^5\right)$$
$$= \frac{2187}{4096}$$

Q9
$$\lim_{x\to 7} \frac{3\sin(x-7)}{(x-7)(x-2)}$$
$$= \lim_{x\to 7} \frac{\sin(x-7)}{x-7} \times \frac{3}{(x-2)}$$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$$= 1 \times \frac{3}{7-2}$$
$$= \frac{3}{5}$$

Q10 a) Ball lands when $y=0$
$$0 = Vt\sin\theta - \frac{g}{2}t^2$$
$$0 = t\left(V\sin\theta - \frac{g}{2}t\right)$$
$$t = 0$$

$t=0$ is when the ball launches so we can ignore it.

$$V\sin\theta = \frac{g}{2}t$$
$$t = \frac{2V\sin\theta}{g}$$
$$x = Vt\cos\theta$$
$$x = V\left(\frac{2V\sin\theta}{g}\right)\cos\theta$$

$2\sin\theta\cos\theta = \sin 2\theta$

$$= \frac{V^2\sin 2\theta}{g}$$

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b) Find cartesian equation by subbing out parameter t.
$$t = \frac{x}{V\cos\theta}$$
$$\therefore y = V\left(\frac{x}{V\cos\theta}\right)\sin\theta - \frac{g}{2}\left(\frac{x}{V\cos\theta}\right)^2$$
$$y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$$
$$= x\tan\theta\left(1 - \frac{gx}{2V^2\cos^2\theta\tan\theta}\right)$$
$$= x\tan\theta\left(1 - \frac{gx}{2V^2\sin\theta\cos\theta}\right)$$

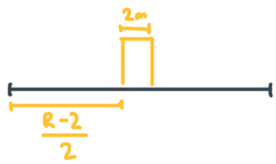
This step is tricky. Be wary of the $\tan\theta$

$$\frac{1}{R} = \frac{g}{V^2\sin 2\theta}$$
$$\therefore y = x\left(1 - \frac{x}{R}\right)\tan\theta$$

c) $\theta = 45°$ $y = 8$
$$8 = x\left(1 - \frac{x}{R}\right) \times 1$$
$$8 = x - \frac{x^2}{R}$$
$$x^2 - Rx + 8R = 0$$

Sub into equation in part(b)

d) First x is at $x = \frac{R-2}{2}$



Using quad formula on the equation in part(c):
$$x = \frac{R \pm \sqrt{R^2 - 4\times1\times8R}}{2}$$
This must equal $\frac{R-2}{2}$ as it's one of the x values:
$$\frac{R - \sqrt{R^2-32R}}{2} = \frac{R-2}{2}$$

notice we don't need the $\pm$ but only a $-$ because we are dealing with the lower x-value

$$\therefore \sqrt{R^2-32R} = 2$$
$$R^2 - 32R = 4$$
$$R^2 - 32R - 4 = 0$$
$$R = \frac{32 \pm \sqrt{32^2 + 4\times1\times4}}{2\times1}$$
$$= \frac{32 \pm 32.12\ldots}{2}$$
$$= 32\text{m (nearest metre)}$$

If you need any of these answers explained further, contact 04Thinking (0484 465 464) and speak with a maths tutor now!